Evolutionary game theory and cognition

Artem Kaznatcheev

University of Waterloo

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Two player games

- A game between two players (Alice and Bob) is represented by a matrix G of pairs.

Example

$$\begin{pmatrix} (3,1) & (2,3) \\ (-1,2) & (3,-1) \end{pmatrix}$$

- If Alice plays strategy i and Bob plays strategy j then $(a,b) := G_{ij}$ is the outcome, where a corresponds to the change in Alice's utility and b to Bob's.

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Zero-sum games

Definition

A game G is a zero-sum game if for each $(a, b) := G_{ij}$ we have $a + b = 0$.

Example

$$\begin{pmatrix} (1,-1) & (-1,1) \\ (-1,1) & (1,-1) \end{pmatrix}$$

- Zero-sum games are the epitome of competition. Any gain for Alice is a loss for Bob, and vice-versa.

Coordination games

Definition

A two-strategy game G is a coordination game if we have

$$G = \begin{pmatrix} (a_1, b_1) & (c_2, d_1) \\ (c_1, d_2) & (a_2, b_2) \end{pmatrix}$$

And $a_1 > c_1$, $a_2 > c_2$, $b_1 > d_1$, $b_2 > d_2$.

Examples

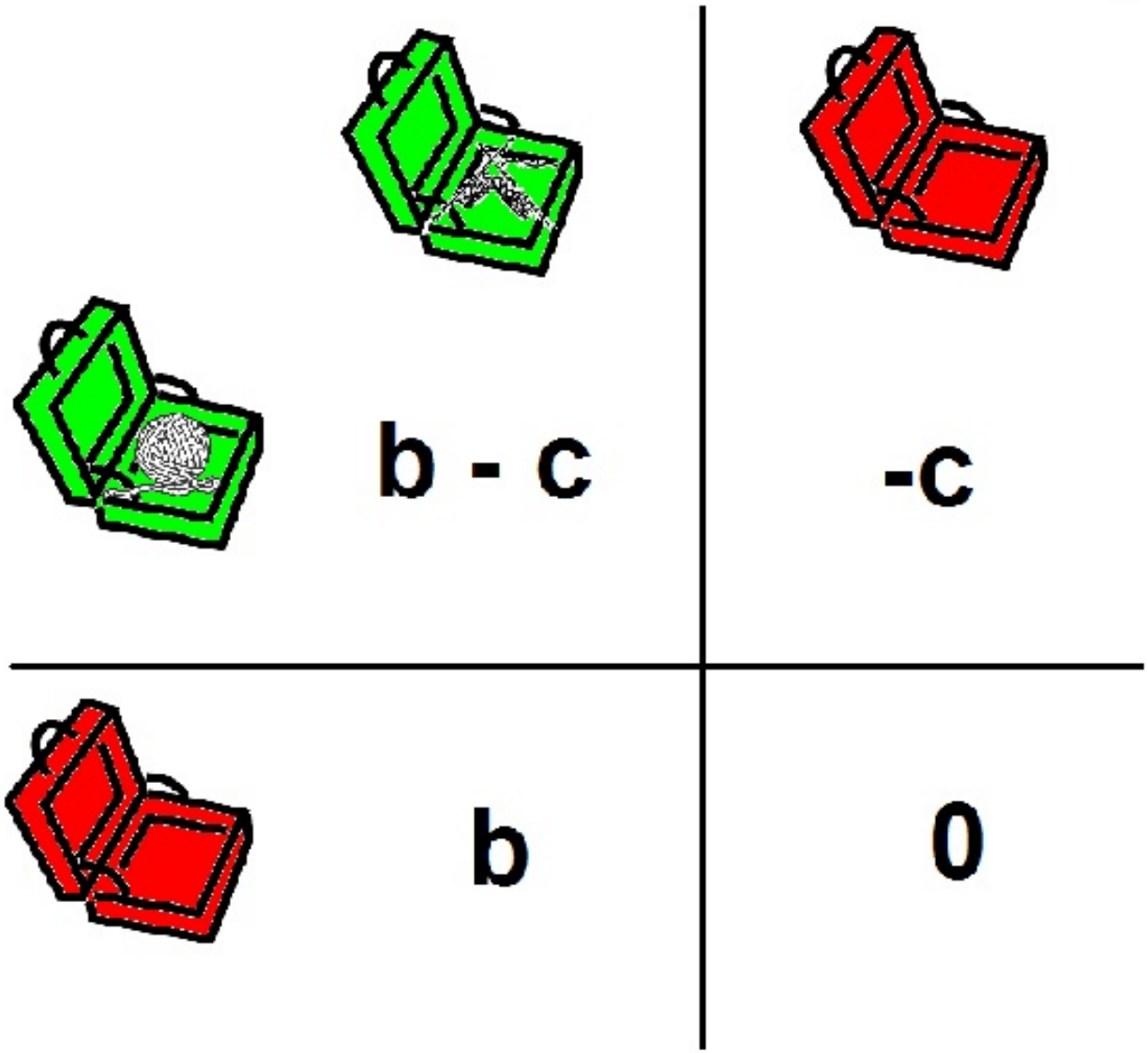
$$\begin{pmatrix} (1,1) & (-1,-1) \\ (-1,-1) & (1,1) \end{pmatrix}, \begin{pmatrix} (2,1) & (0,0) \\ (0,0) & (1,2) \end{pmatrix}, \begin{pmatrix} (4,4) & (0,2) \\ (2,0) & (3,3) \end{pmatrix}$$

- The diagonals are always better for both players, they just have to figure out how to pick the same strategy.
- Captures the idea of win-win, lose-lose situations.

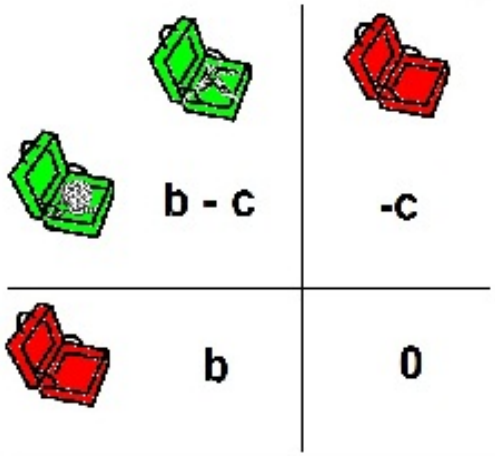
What do these two types of games tell us?

- Zero-sum and coordination games are mutually exclusive: there is no game that is both zero-sum and a coordination game.
- Upside: zero-sum and coordination provide a good duality between impossibility of cooperation and obvious cooperation.
- Downside: both types of games are really boring. The most interesting games (from a mathematical and modeling point of view) are neither zero-sum nor coordination.
- Being non-zero-sum does not ensure cooperation.

Prisoner's dilemma



Prisoner's dilemma



$$\begin{pmatrix} (b-c, b-c) & (-c, b) \\ (b, -c) & (0,0) \end{pmatrix}$$

- b is the benefit of receiving and c is the cost of giving.
- Strategy 1 is called cooperate or C and strategy 2 is called defect or D.
- The rational strategy (or Nash equilibrium) is mutual defection.
- The best for the players taken together (or Pareto optimum) is mutual cooperation.

Nash equilibrium

Definition

A strategy pair (p, q) is a Nash equilibrium of a game G if for all other strategies r we have:

$$fst(G(p, q)) \geq fst(G(r, q))$$

and

$$snd(G(p, q)) \geq snd(G(p, r))$$

- Informally: neither Alice nor Bob can improve their payoff by unilateral change of strategy.
- If we only allow pure strategies then replace $G(i, j)$ by G_{ij}
- If we allow mixed strategies, then every game has at least one Nash equilibrium

Pareto optimum

Definition

A strategy pair (p, q) is a Pareto optimum of a game G is there is no other strategy pair (p', q') such that $G(p', q') > G(p, q)$

- Informally: there is no other strategy such that both Alice and Bob get a better payoff.
- Every game has at least one Pareto optimum

Example

$$\begin{pmatrix} (2,1) & (0,0) \\ (0,0) & (1,2) \end{pmatrix}, \begin{pmatrix} (2,-3) & (-1,1) \\ (0,0) & (-2,2) \end{pmatrix}$$

Cognitive demands of rationality

- Alice needs to be aware of her own utility function
- To check if she is currently in Nash equilibrium (at least from her perspective) Alice needs to be able to simulate the game in her mind (thus she must understand the interaction)
- To find a Nash equilibrium Alice needs to be able to simulate the game and she must be able to place herself in Bob's shoes.
- Do we even expect humans to be able to do all of this?
- Let's bound rationality and see what happens!

Evolutionary game theory

- Strategy is a genetic trait and immutable by the agent.
- All cognition is stripped away
- Game payoffs change the fitness of the agent.
- Agents reproductive rate increases with higher fitness.
- Simplest model of biological evolution.
- Also applicable outside of biology.
- What happens to rationality?

Evolutionary stable strategy

Definition

A strategy s is an evolutionary stable strategy for a game G if for all other strategies r we have (a) $fst(G(s,s)) > fst(G(r,s))$, or (b) $fst(G(s,s)) = fst(G(r,s))$ and $fst(G(s,r)) > fst(G(r,r))$.

- Consider a population all with strategy s, a mutant with strategy r can invade the population only if one of the following conditions holds:
  - r has a higher fitness than s in a population of all s.
  - r has the same fitness when interacting with s and the same or greater fitness when interacting with other r.
- Compare this to the Nash equilibrium conditions.

ESS vs. Nash

Definition

A strategy s is an evolutionary stable strategy for a game G if for all other strategies r we have (a) $fst(G(s,s)) > fst(G(r,s))$, or (b) $fst(G(s,s)) = fst(G(r,s))$ and $fst(G(s,r)) > fst(G(r,r))$.

Definition

A strategy s is a Nash equilibrium strategy of a game G if for all other strategies r we have (a) $fst(G(s,s)) \geq fst(G(r,s))$ and (b) $snd(G(s,s)) \geq snd(G(s,r))$.

- Most evolutionary games are symmetric games, so $fst(G(r,s)) = snd(G(s,r))$ and $fst(G(s,s)) = snd(G(s,s))$.
- The conditions are almost identical: we can think of the evolutionary process as a rational process (entity?)!.

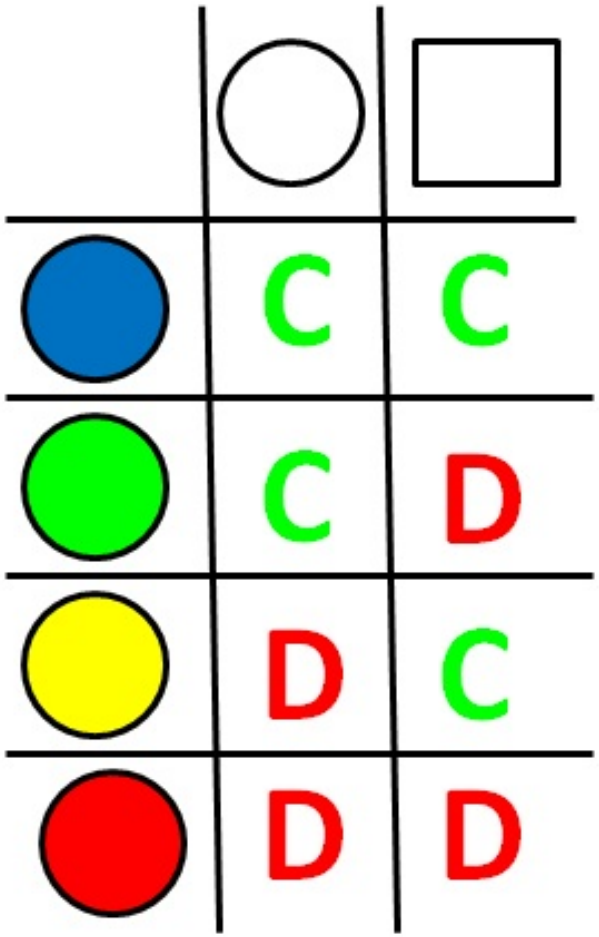
Wait a second: what about cooperation?

- The ESS predicts mutual defection in the Prisoner's dilemma, but we observe cooperation through out nature.
- The assumptions of the ESS:
  - Random interactions (inviscid population)
  - No repeated interactions
  - Zero cognition in individual agents
- Various augmentations of the model create fascinating results, among them: cooperation.

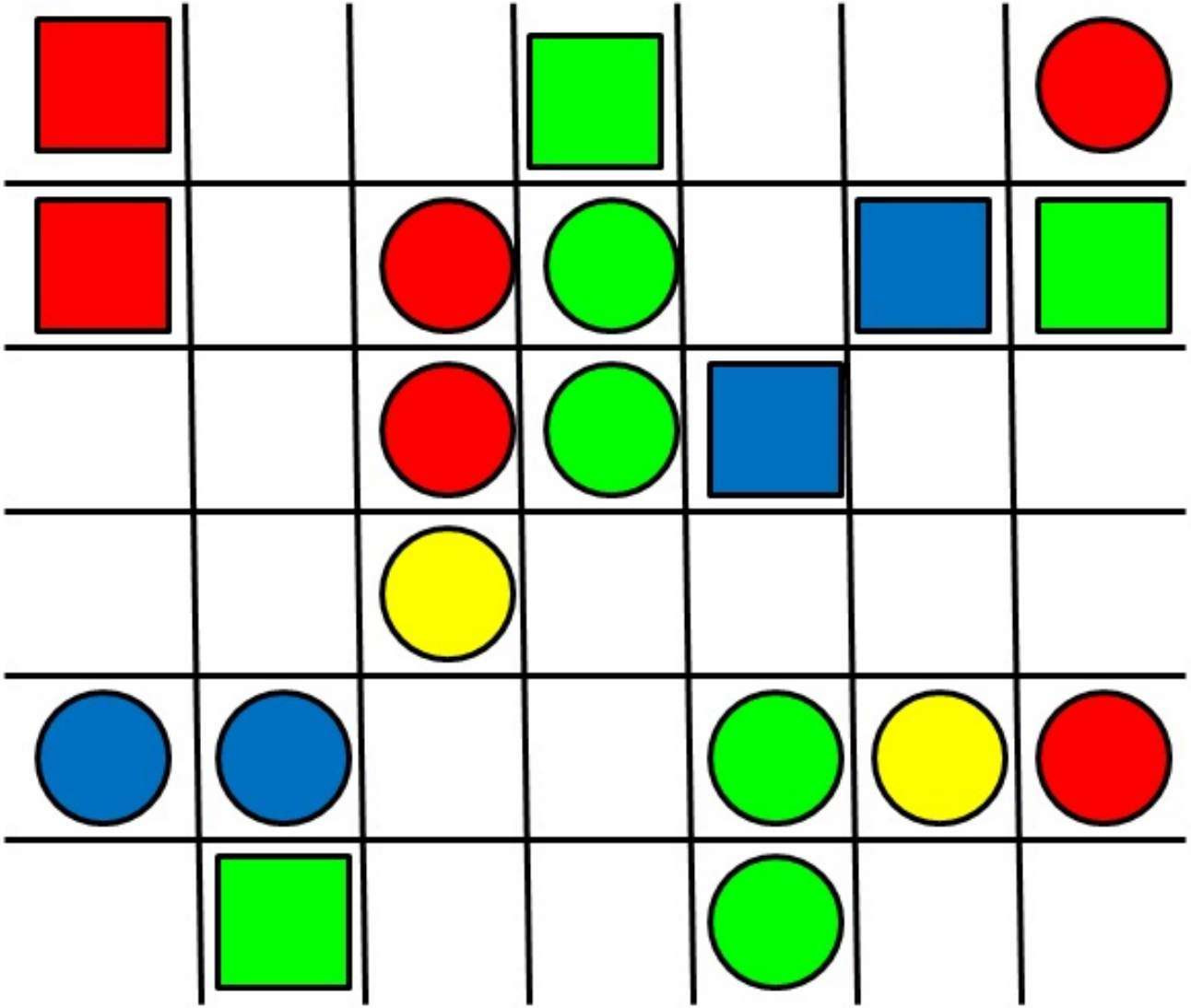
Cognitively relevant augmentations

- Kin selection: the ability to recognize your children, siblings and parents
- Direct reciprocity (reciprocal altruism): the ability to remember previous interactions
- Indirect reciprocity: the ability to track social constructs like reputation
- Tag-based conditional strategies

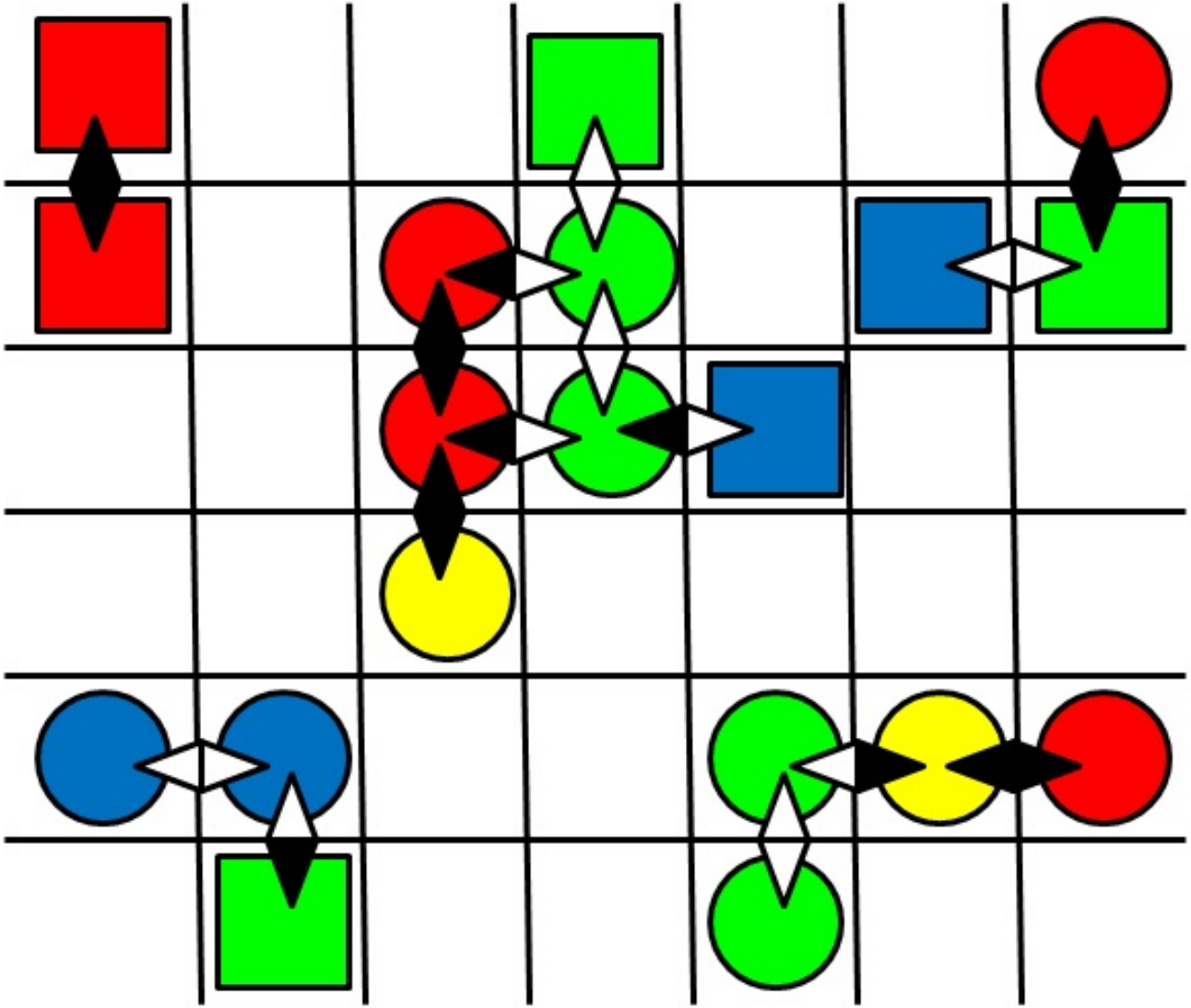
Ethnocentrism in spatial models



Ethnocentrism in spatial models



Ethnocentrism in spatial models



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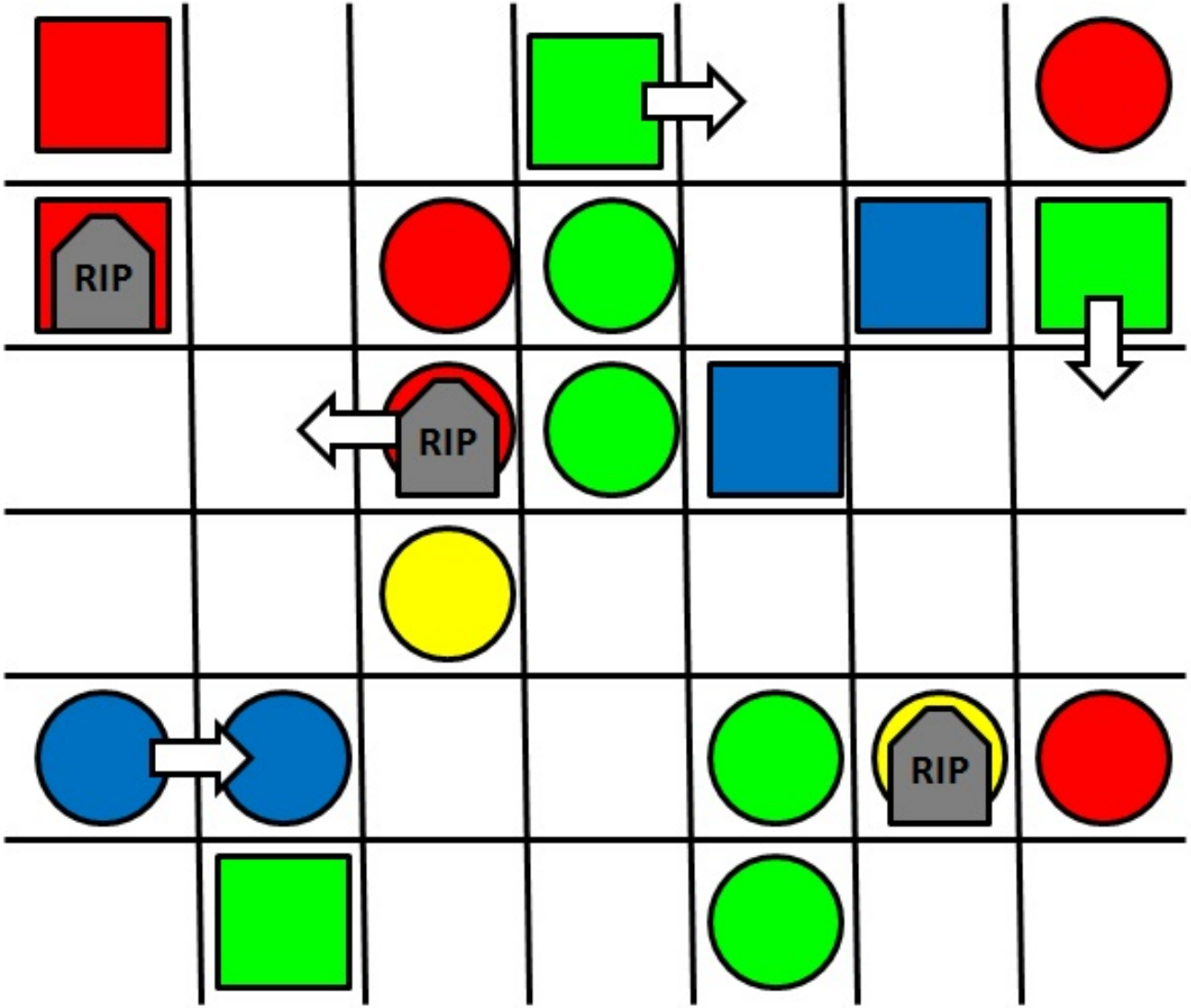

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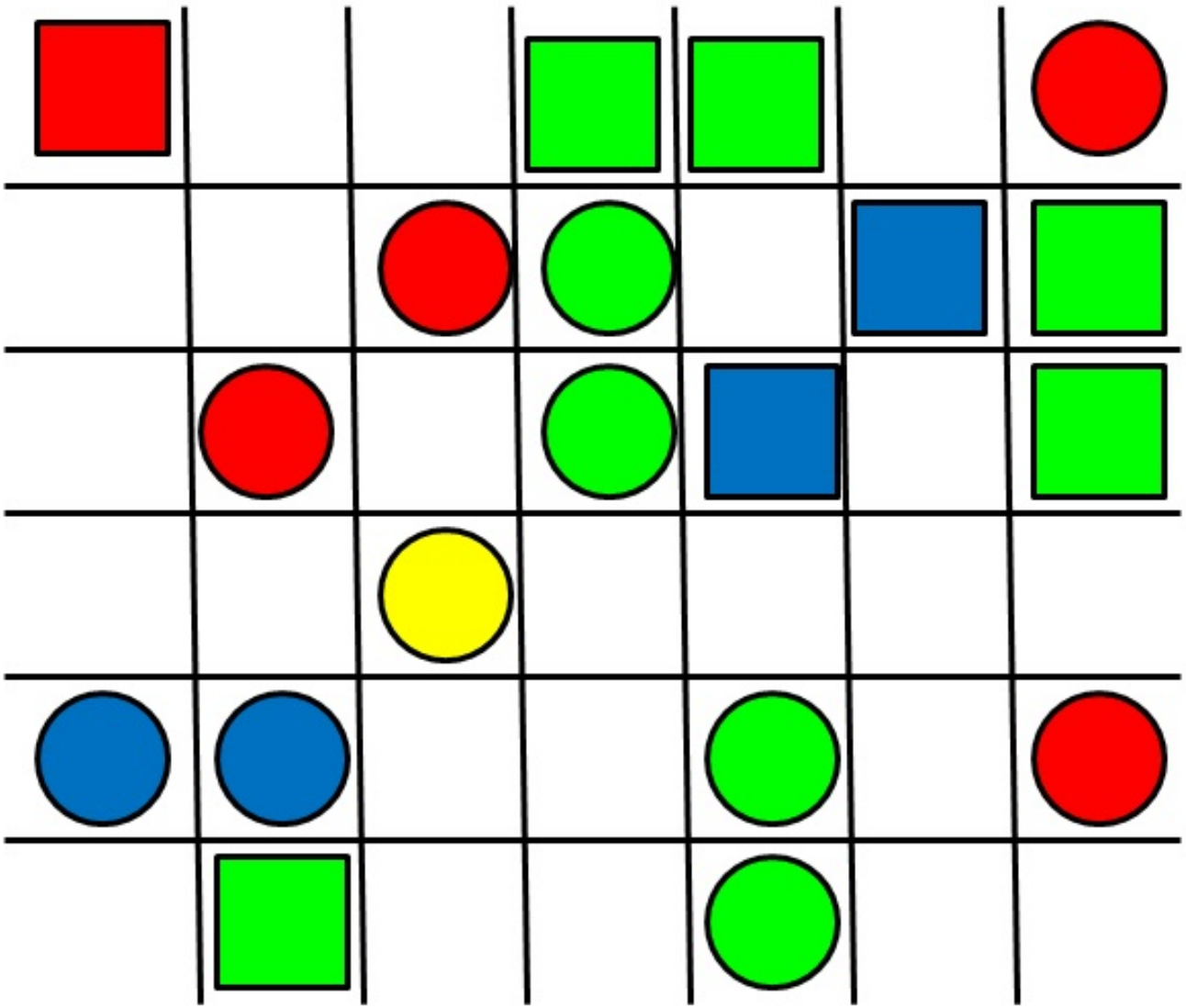

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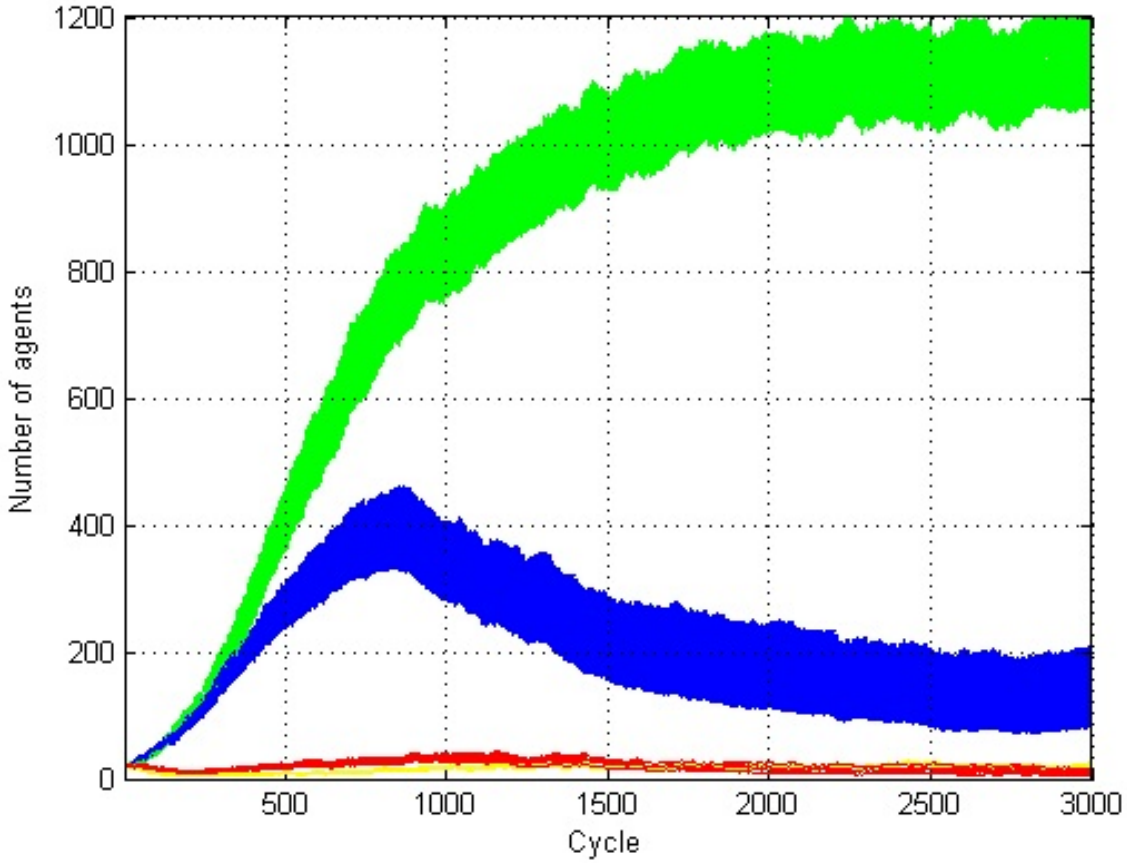
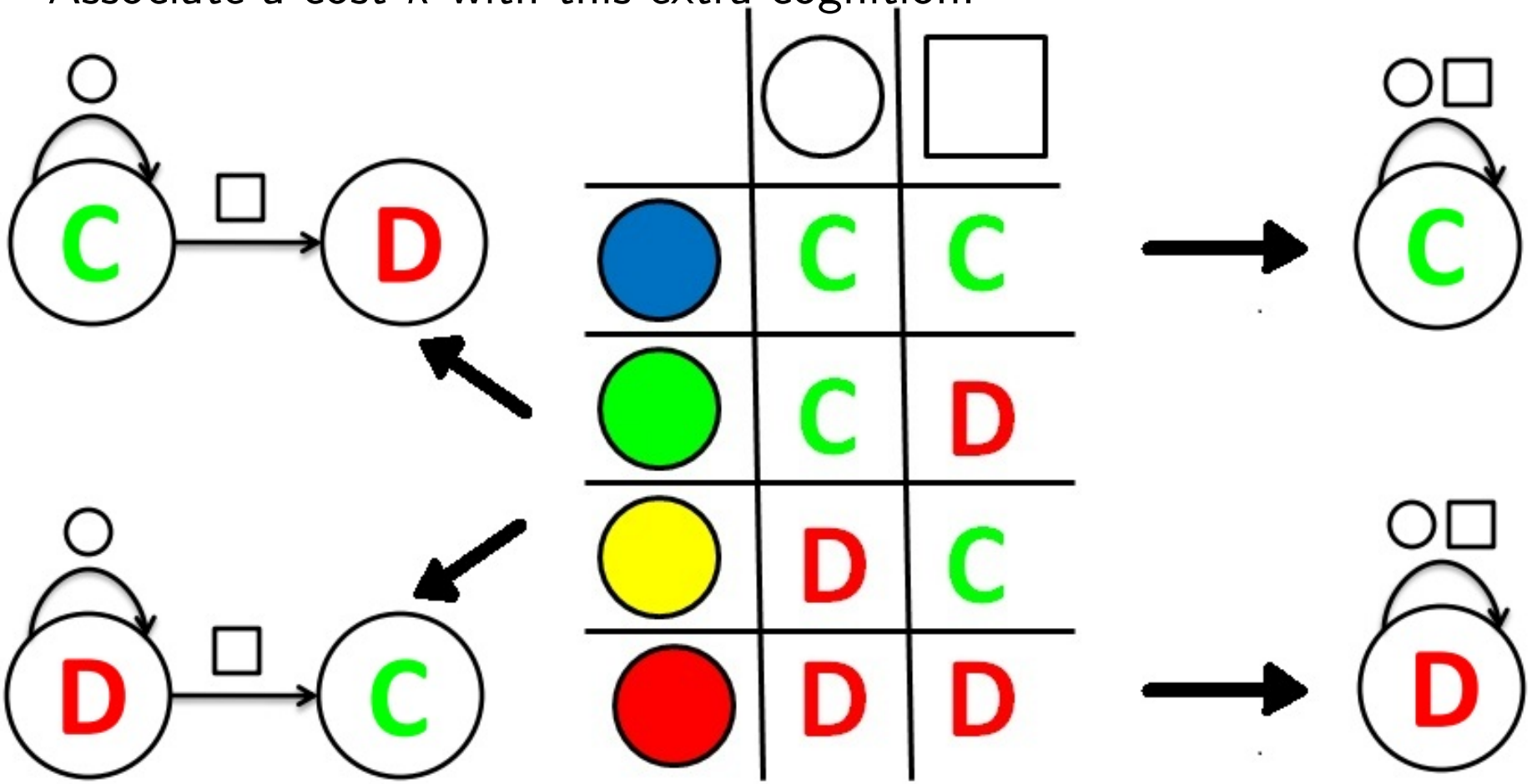
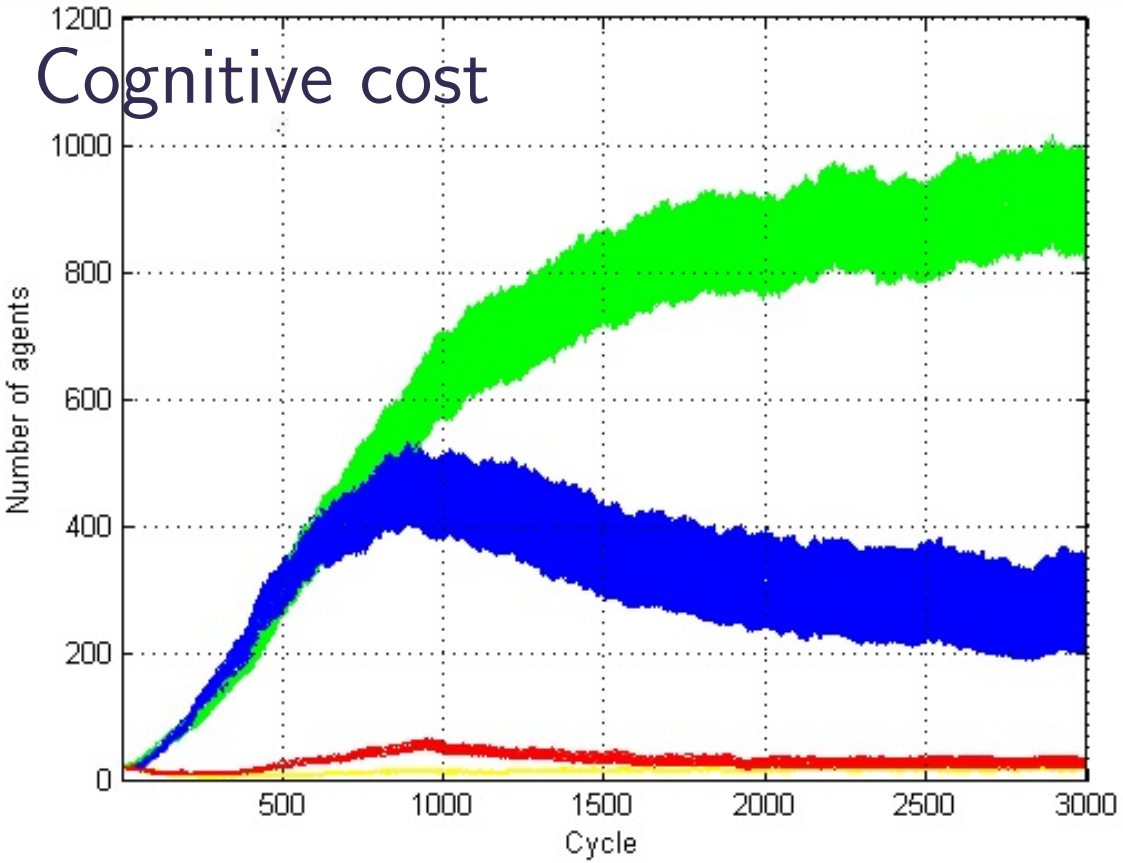


Cognitive cost

Associate a cost k with this extra cognition.

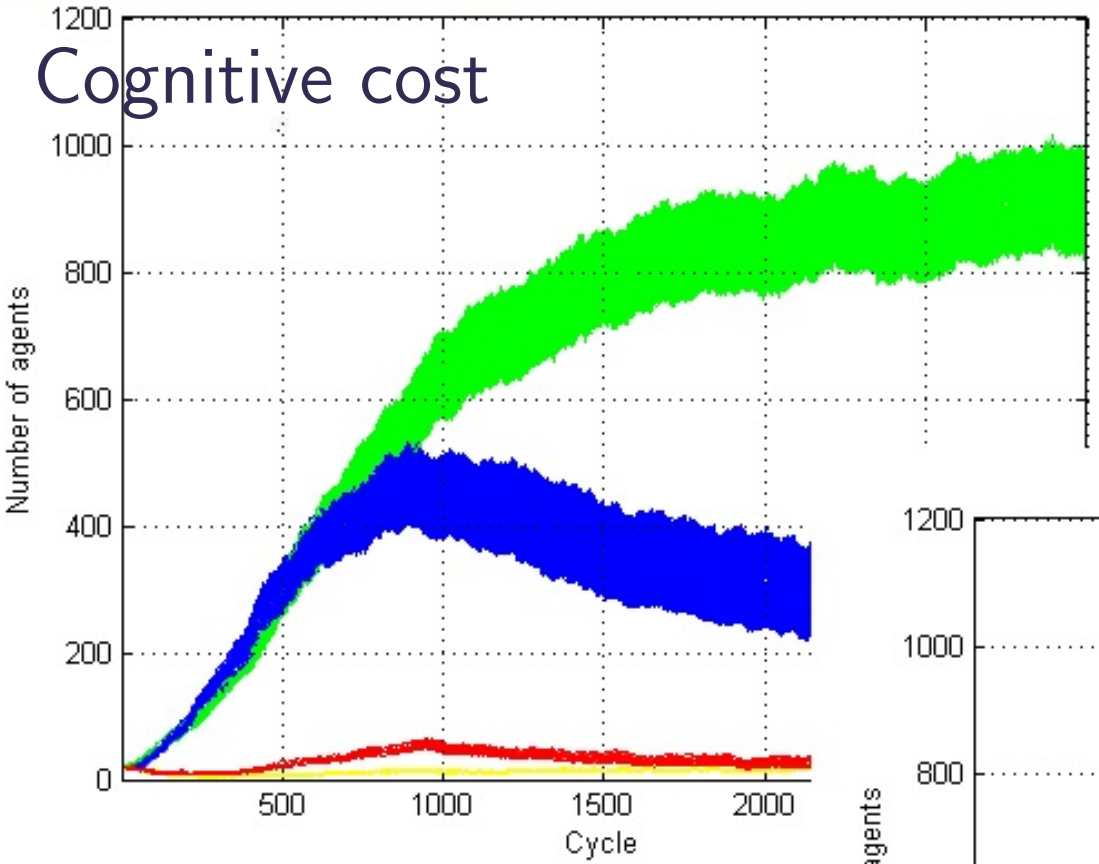


Cognitive cost



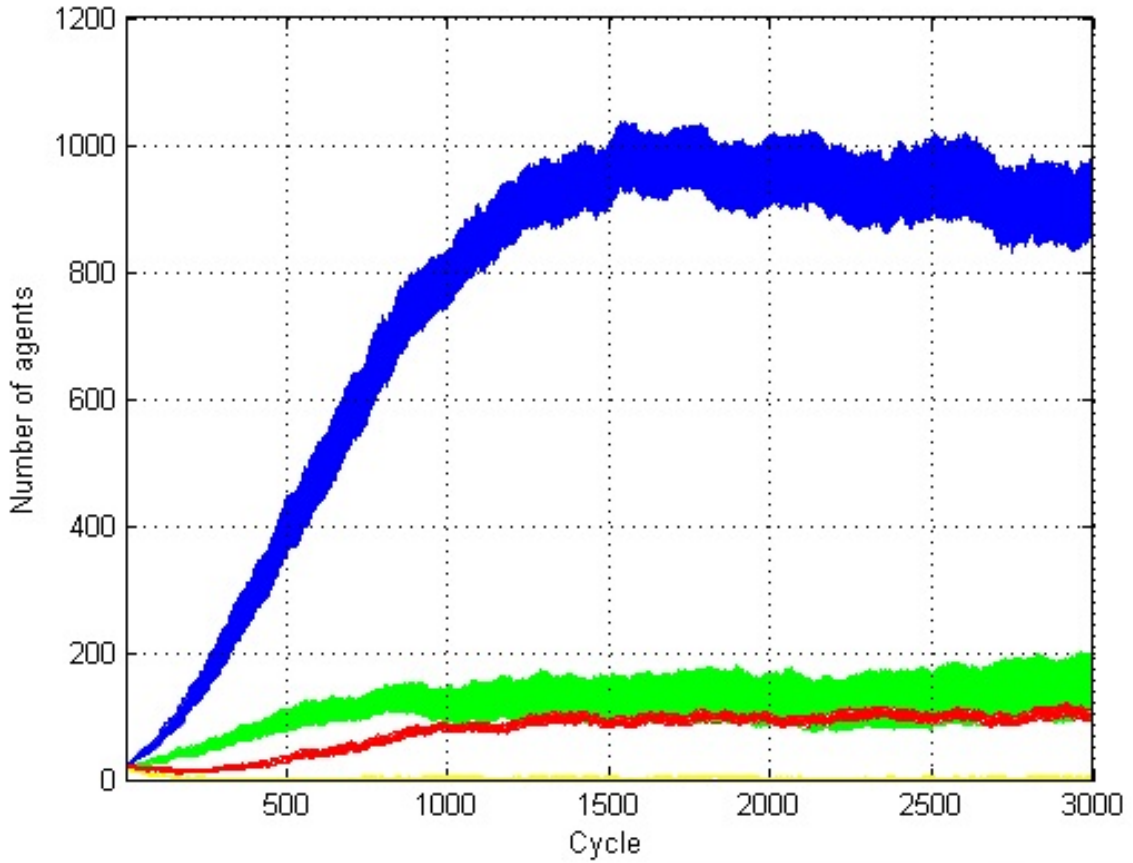
k = 0.02

Cognitive cost

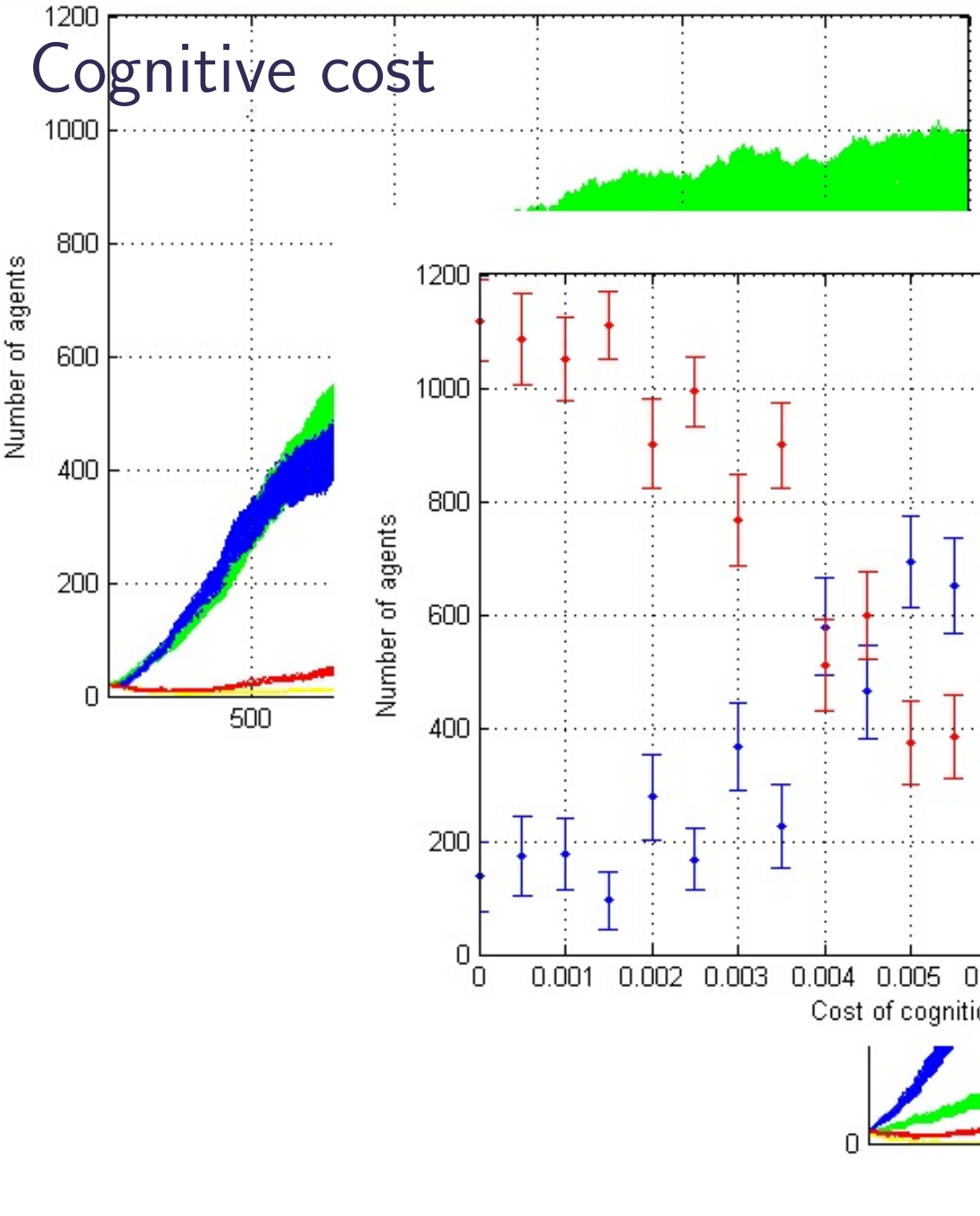


k = 0.02

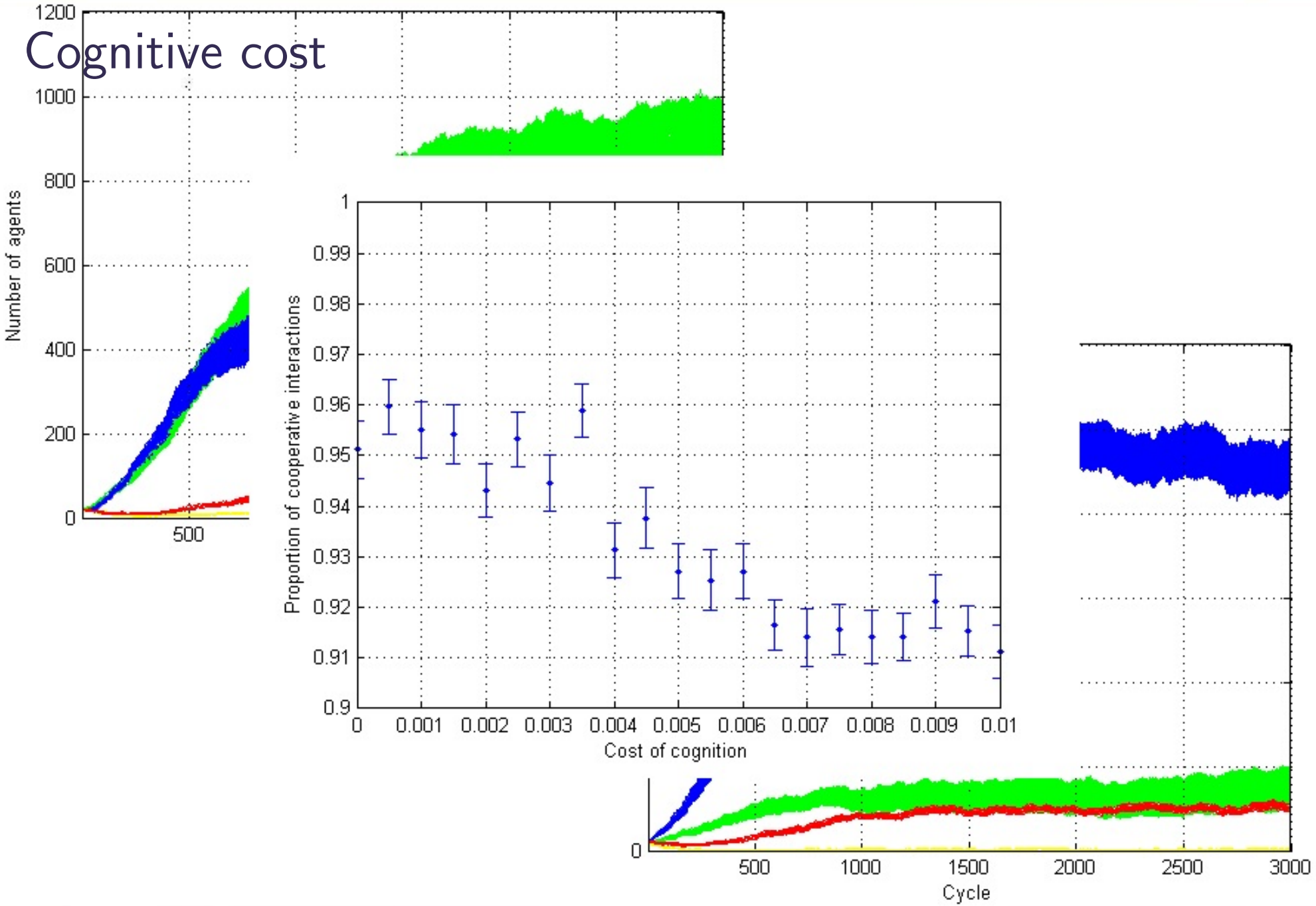
k = 0.07



Cognitive cost



Cognitive cost



Can I learn more?

1. Evolution of ethnocentrism:
   - T.R. Shultz, M. Hartshorn, and AK. [2009] "Why is ethnocentrism more common than humanitarianism?" Proceedings of the 31st annual conference of the cognitive science society.
   - AK, and T.R. Shultz. [2011] "Ethnocentrism Maintains Cooperation, but Keeping One's Children Close Fuels It." Proceedings of the 33rd annual conference of the cognitive science society.
2. Cognitive cost of ethnocentrism:
   - AK. [2010] "The cognitive cost of ethnocentrism." Proceedings of the 32nd annual conference of the cognitive science society.
3. General work on EGT:
   - M.A. Nowak [2006] "Evolutionary Dynamics". Reading suggestions: `http://egtheory.wordpress.com/2011/09/05/nowak-evolutionary-dynamics/`
   - Selected reading from the literature: `http://egtheory.wordpress.com/2011/09/01/previously-rea/`

Can I get involved?

The literature is pretty extensive, it is best to seek guidance when picking a project. The evolutionary games group blog is one such resource:

1. Evolution of perception and deception
   `http://egtheory.wordpress.com/2011/09/19/perception-deception/`
2. Ethnocentrism with probabilistic strategies
   `http://egtheory.wordpress.com/2011/09/26/probabilistic-strategies/`
3. Cognitive cost of agency
   `http://egtheory.wordpress.com/2011/10/03/cognitive-cost-of-agency/`
4. Julian Xue's Irreversible evolution
   `http://egtheory.wordpress.com/2011/10/06/irreversible-evolution/`

Thank you!

For more info feel free to contact me at:

artem.kaznatcheev@mail.mcgill.ca

Some fun resources:

1. Robert Wright: "The evolution of compassion"
   http://www.ted.com/talks/lang/eng/robert_wright_the_evolution_of_compassion.html
2. Howard Rheingold: "On collaboration"
   http://www.ted.com/talks/lang/eng/howard_rheingold_on_collaboration.html
3. Jonathan Haidt: "On the moral roots of liberals and conservatives"
   http://www.ted.com/talks/jonathan_haidt_on_the_moral_mind.html
4. Artem Kaznatcheev: "Evolving Cooperation"
   http://www.youtube.com/watch?v=bRuE3oP-JT8
5. Stanford Encyclopedia of Philosophy: "Evolutionary Game Theory"
   http://plato.stanford.edu/entries/game-evolutionary/